Section 4.9 –Trigonometric Identities

Important Ideas:

Reciprocal Identities:	Pythagorean Identities	Quotient Identities
$\csc\theta = \frac{1}{\sin\theta}$	$\sin^2\theta + \cos^2\theta = 1$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\sec\theta = \frac{1}{\cos\theta}$	$\cot^2\theta + 1 = \csc^2\theta$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\cot\theta = \frac{1}{\tan\theta}$	$\tan^2\theta + 1 = \sec^2\theta$	

Check Your Understanding!

1. If $\sec\theta = \frac{7}{2}$, find $\cos\theta$, $\tan\theta$, $\sin\theta$, $\csc\theta$, and $\cot\theta$.

For questions 2-6, simplify each trig expression to one number or one trig expression.

2. $\tan\theta \cdot \cot\theta =$

3. $\sec^2\theta\,(1 - \sin^2\theta) =$

4. $\frac{\cos^2 x + \sin^2 x}{\sec x} =$

5. $\sec^2 x - \tan^2 x =$

6. **Challenge!** $\csc x - \cos x \cot x =$

Guidelines for Verifying Trigonometric Identities

1. **Work with one side of the equation at a time**. It is often better to work with the more complicated side first.
2. **Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator**.
3. **Look for opportunities to use the fundamental identities** (see box on reverse side). Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents, and cosecants and cotangents.
4. **When the preceding guidelines do not help, try converting all terms to sines and cosines**.
5. *Always try something*. Even making an attempt that leads to a dead end provides insight.

Complete the identity.

1. $\dfrac{1}{\tan u} = \_\_\_\_$

2. $\dfrac{1}{\csc u} = \_\_\_\_$

3. $\dfrac{\sin u}{\cos u} = \_\_\_\_$

4. $\dfrac{1}{\sec u} = \_\_\_\_$

5. $\sin^2 u + \_\_\_\_ = 1$

Fill in the missing step.

33. $\sec^4 x - 2\sec^2 x + 1 = (\sec^2 x - 1)^2$

$$= \boxed{\quad}$$

$$= \tan^4 x$$

Verify the identity.

11. $\sin t \csc t = 1$

12. $\sec y \cos y = 1$

13. $\dfrac{\csc^2 x}{\cot x} = \csc x \sec x$

14. $\dfrac{\sin^2 t}{\tan^2 t} = \cos^2 t$

15. $\cos^2 \beta - \sin^2 \beta = 1 - 2\sin^2 \beta$

16. $\cot^2 \beta + \csc^2 \beta = 2\csc^2 \beta - 1$

17. $\tan^2 \theta + 6 = \sec^2 \theta + 5$

18. $3 + \sin^2 z = 4 - \cos^2 z$

19. $(1 + \sin x)(1 - \sin x) = \cos^2 x$

Fill in the missing steps.

34. $$\frac{\tan x - \cot x}{\tan x + \cot x} = \frac{\dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\sin x}}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}$$

$$= \boxed{\quad}$$

$$= \frac{\sin^2 x - \cos^2 x}{1}$$

$$= \sin^2 x - \cos^2 x$$

$$= \boxed{\quad}$$

$$= 1 - 2\cos^2 x$$

Describe the error.

31.

$$(1 + \tan x)[1 + \cot(-x)] = (1 + \tan x)(1 + \cot x)$$

$$= 1 + \cot x + \tan x + \tan x \cot x$$

$$= 1 + \cot x + \tan x + 1$$

$$= 2 + \cot x + \tan x$$

Verify the identity.

41. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$

42. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$