1.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

For instructions on how to use a graphing utility, see Appendix A.

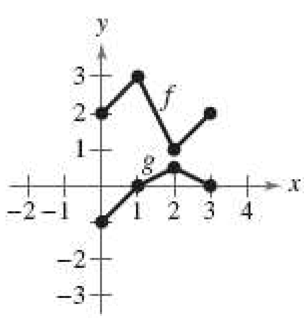
Vocabulary and Concept Check

In Exercises 1, 2, 3, and 4, fill in the blank(s).

1. Two functions f and g can be combined by the arithmetic operations of ____ , ____ , ____ , and ____ to create new functions.
2. The ____ of the function f with the function g is $(f \circ g)(x) = f(g(x))$.
3. The domain of $f \circ g$ is the set of all x in the domain of g such that ____ is in the domain of f.
4. To decompose a composite function, look for an ____ and an ____ function.
5. Given $f(x) = x^2 + 1$ and $(fg)(x) = 2x(x^2 + 1)$, what is $g(x)$?
6. Given $(f \circ g)(x) = f(x^2 + 1)$, what is $g(x)$?

Procedures and Problem Solving

Graphing the Sum of Two Functions In Exercises 7, 8, 9, and 10, use the graphs of f and g to graph $h(x) = (f + g)(x)$ To print an enlarged copy of the graph, go to MathGraphs.com.

7. 

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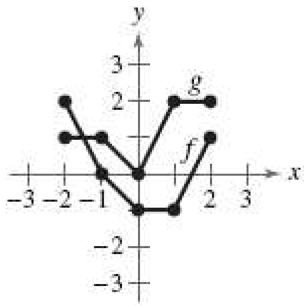

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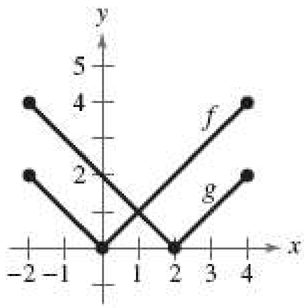

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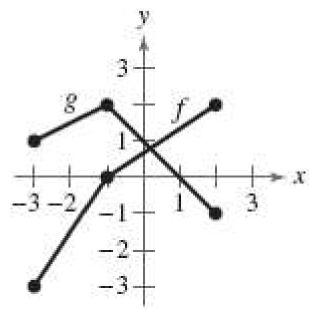


Finding Arithmetic Combinations of Functions In Exercises 11, 12, 13, 14, 15, 16, 17, and 18, find

a. $(f+g)(x)$,

b. $(f-g)(x)$,

c. $(fg)(x)$, and

d. $(f/g)(x)$.

What is the domain of f/g?

11. $f(x)=x+3, \quad g(x)=x-3$

12. $f(x)=2x-5, \quad g(x)=1-x$

13. $f(x)=3x^2, \quad g(x)=6-5x$

14. $f(x)=2x+5, \quad g(x)=x^2-9$

15. $f(x)=x^2+5, \quad g(x)=\sqrt{1-x}$

16. $f(x)=\sqrt{x^2-4}, \quad g(x)=\dfrac{x^2}{x^2+1}$

17. $f(x)=\dfrac{1}{x}, \quad g(x)=\dfrac{1}{x^2}$

18. $f(x)=\dfrac{x}{x+1}, \quad g(x)=\dfrac{1}{x^3}$

Evaluating an Arithmetic Combination of Functions In Exercises 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, and 32, evaluate the indicated function for $f(x)=x^2-1$ and $g(x)=x-2$ algebraically. If possible, use a graphing utility to verify your answer.

19. $(f+g)(3)$

20. $(f-g)(-2)$

21. $(f-g)(0)$

22. $(f+g)(1)$

23. $(fg)(-6)$

24. $(fg)(4)$

25. $(f/g)(-5)$

26. $(f/g)(0)$

27. $(f-g)(t+1)$

28. $(f+g)(t-3)$

29. $(fg)(-5t)$

30. $(fg)(3t^2)$

31. $(f/g)(t-4)$

32. $(f/g)(t+2)$

Graphing an Arithmetic Combination of Functions In Exercises 33, 34, 35, and 36, use a graphing utility to graph the functions f, g, and h in the same viewing window.

33. $f(x)=\dfrac{1}{2}x, \quad g(x)=x-1, \quad h(x)=f(x)+g(x)$

34. $f(x)=\dfrac{1}{3}x, \quad g(x)=-x+4, \quad h(x)=f(x)-g(x)$

35. $f(x) = x^2, \quad g(x) = -2x + 5, \quad h(x) = f(x) \cdot g(x)$

36. $f(x) = 4 - x^2, \quad g(x) = x, \quad h(x) = f(x)/g(x)$

Graphing a Sum of Functions In Exercises 37, 38, 39, and 40, use a graphing utility to graph f, g, and $f + g$ in the same viewing window. Which function contributes most to the magnitude of the sum when $0 \leq x \leq 2$? Which function contributes most to the magnitude of the sum when $x > 6$?

37. $f(x) = 3x, \quad g(x) = -\frac{x^3}{10}$

38. $f(x) = \frac{x}{2}, \quad g(x) = \sqrt{x}$

39. $f(x) = 3x + 2, \quad g(x) = -\sqrt{x + 5}$

40. $f(x) = x^2 - \frac{1}{2}, \quad g(x) = -3x^2 - 1$

Compositions of Functions In Exercises 41, 42, 43, and 44, find

a. $f \circ g$,

b. $g \circ f$ and, if possible,

c. $(f \circ g)(0)$.

41. $f(x) = 2x^2, \quad g(x) = x + 4$

42. $f(x) = \sqrt[3]{x - 1}, \quad g(x) = x^3 + 1$

43. $f(x) = 3x + 5, \quad g(x) = 5 - x$

44. $f(x) = x^3, \quad g(x) = \frac{1}{x}$

Finding the Domain of a Composite Function In Exercises 45, 46, 47, 48, 49, 50, 51, 52, 53, and 54, determine the domains of

a. f,

b. g, and

c. $f \circ g$.

Use a graphing utility to verify your results.

45. $f(x) = \sqrt{x - 7}, \quad g(x) = 4x^2$

46. $f(x) = \sqrt{x + 3}, \quad g(x) = \frac{x}{2}$

47. $f(x) = x^2 + 1, \quad g(x) = \sqrt{x}$

48. $f(x) = x^{1/4}, \quad g(x) = x^4$

49. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x+3}$

50. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{2x}$

51. $f(x) = |x - 4|, \quad g(x) = 3 - x$

52. $f(x) = \dfrac{2}{|x|} \quad g(x) = x - 5$

53. $f(x) = x + 2, \quad g(x) = \dfrac{1}{x^2 - 4}$

54. $f(x) = \dfrac{3}{x^2 - 1}, \quad g(x) = x + 1$

Determining Whether $f \circ g = g \circ f$ In Exercises 55, 56, 57, 58, 59, and 60,

a. find $f \circ g$, $g \circ f$, and the domain of $f \circ g$.

b. Use a graphing utility to graph $f \circ g$ and $g \circ f$.

Determine whether $f \circ g = g \circ f$.

55. $f(x) = \sqrt{x+4}, \quad g(x) = x^2$

56. $f(x) = \sqrt[3]{x+1}, \quad g(x) = x^3 - 1$

57. $f(x) = \dfrac{1}{3}x - 3, \quad g(x) = 3x + 9$

58. $f(x) = \sqrt{x}, \quad g(x) = \sqrt{x}$

59. $f(x) = x^{2/3}, \quad g(x) = x^6$

60. $f(x) = |x|, \quad g(x) = -x^2 + 1$

Determining Whether $f \circ g = g \circ f$ In Exercises 61, 62, 63, 64, 65, and 66,

a. find $(f \circ g)(x)$ and $(g \circ f)(x)$,

b. determine algebraically whether $(f \circ g)(x) = (g \circ f)(x)$, and

c. use a graphing utility to complete a table of values for the two compositions to confirm your answer to part (b).

61. $f(x) = 5x + 4, \quad g(x) = \frac{1}{5}(x - 4)$

62. $f(x) = \frac{1}{4}(x - 1), \quad g(x) = 4x + 1$

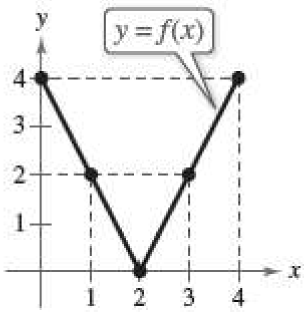
63. $f(x) = \sqrt{x + 6}, \quad g(x) = x^2 - 5$

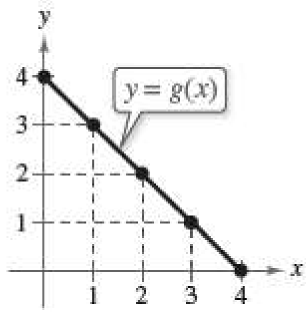
64. $f(x) = x^3 - 4, \quad g(x) = \sqrt[3]{x + 10}$

65. $f(x) = |x|, \quad g(x) = 2x^3$

66. $f(x) = \frac{6}{3x - 5}, \quad g(x) = -x$

Evaluating Combinations of Functions In Exercises 67, 68, 69, and 70, use the graphs of f and g to evaluate the functions.





67.

a. $(f + g)(3)$

b. $(f/g)(2)$

68.

a. $(f - g)(1)$

b. $(fg)(4)$

69.

a. $(f \circ g)(3)$

b. $(g \circ f)(2)$

70.

a. $(f \circ g)(1)$

b. $(g \circ f)(3)$

Identifying a Composite Function In Exercises 71, 72, 73, 74, 75, 76, 77, and 78, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

71. $h(x) = (2x+1)^2$

72. $h(x) = (1-x)^3$

73. $h(x) = \sqrt[3]{x^2-4}$

74. $h(x) = \sqrt{9-x}$

75. $h(x) = \dfrac{1}{x+2}$

76. $h(x) = \dfrac{4}{(5x+2)^2}$

77. $h(x) = (x+4)^2 + 2(x+4)$

78. $h(x) = (x+3)^{3/2} + 4(x+3)^{1/2}$

79. Why You Should Learn It *(*1.5 Combinations of Functions*)* The research and development department of an automobile manufacturer has determined that when required to stop quickly to avoid an accident, the distance (in feet) a car travels during the driver's reaction time is given by

$$R(x) = \frac{3}{4}x$$

where x is the speed of the car in miles per hour. The distance (in feet) traveled while the driver is braking is given by

$$B(x) = \frac{1}{15}x^2.$$

a. Find the function that represents the total stopping distance T.

b. Use a graphing utility to graph the functions R, B, and T in the same viewing window for $0 \leq x \leq 60$.

c. Which function contributes most to the magnitude of the sum at higher speeds? Explain.

80. Modeling Data The table shows the total amounts (in billions of dollars) of health consumption expenditures in the United States (including Puerto Rico) for the years 2002 through 2012. The variables y_1, y_2, and y_3 represent out-of-pocket payments, insurance premiums, and other types of payments, respectively.

(Source: U.S. Centers for Medicare and Medicaid Services)

Year	y_1	y_2	y_3
2002	222	1122	140
2003	238	1223	153
2004	252	1322	159
2005	267	1417	168
2006	277	1521	177
2007	294	1612	187
2008	301	1703	182
2009	301	1799	185
2010	306	1874	195
2011	316	1943	202
2012	328	2014	216

Spreadsheet at LarsonPrecalculus.com

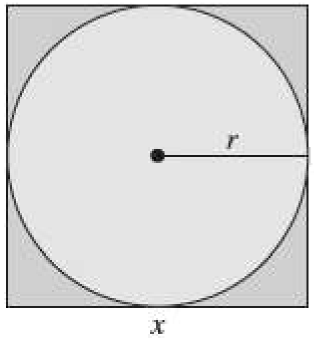
The data are approximated by the following models, where t represents the year, with $t = 2$ corresponding to 2002.

$$y_1 = -0.62t^2 + 18.7t + 188$$
$$y_2 = -1.86t^2 + 116.4t + 890$$
$$y_3 = -0.12t^2 + 8.3t + 128$$

a. Use the models and the *table* feature of a graphing utility to create a table showing the values of y_1, y_2, and y_3 for each year from 2002 through 2012. Compare these models with the original data. Are the models a good fit? Explain.

b. Use the graphing utility to graph y_1, y_2, y_3, and $y_T = y_1 + y_2 + y_3$ in the same viewing window. What does the function y_T represent?

81. Geometry A square concrete foundation was prepared as a base for a large cylindrical gasoline tank (see figure).



a. Write the radius r of the tank as a function of the length x of the sides of the square.

b. Write the area A of the circular base of the tank as a function of the radius r.

c. Find and interpret $(A \circ r)(x)$.

82. Geometry A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in feet) of the outermost ripple is given by $r(t) = 0.6t$, where t is the time (in seconds) after the pebble strikes the water. The area of the circle is given by $A(r) = \pi rt^2$. Find and interpret $(A \circ r)(t)$.

83. Business A company owns two retail stores. The annual sales (in thousands of dollars) of the stores each year from 2009 through 2015 can be approximated by the models

$$S_1 = 973 + 1.3t^2$$

and

$$S_2 = 349 + 72.4t$$

where t is the year, with $t = 9$ corresponding to 2009.

a. Write a function T that represents the total annual sales of the two stores.

b. Use a graphing utility to graph S_1, S_2, and T in the same viewing window.

84. Business The annual cost C (in thousands of dollars) and revenue R (in thousands of dollars) for a company each year from 2009 through 2015 can be approximated by the models

$$C = 254 - 9t + 1.1t^2$$

and

$$R = 341 + 3.2t$$

where t is the year, with $t = 9$ corresponding to 2009.

a. Write a function P that represents the annual profits of the company.

b. Use a graphing utility to graph C, R, and P in the same viewing window.

85. Biology The number of bacteria in a refrigerated food product is given by

$$N(T) = 10T^2 - 20T + 600, \quad 1 \leq T \leq 20$$

where T is the temperature of the food in degrees Celsius. When the food is removed from the refrigerator, the temperature of the food is given by

$$T(t) = 2t + 1$$

where t is the time in hours.

a. Find the composite function $N(T(t))$ or $(N \circ T)(t)$ and interpret its meaning in the context of the situation.

b. Find $(N \circ T)(12)$ and interpret its meaning.

c. Find the time when the bacteria count reaches 1200.

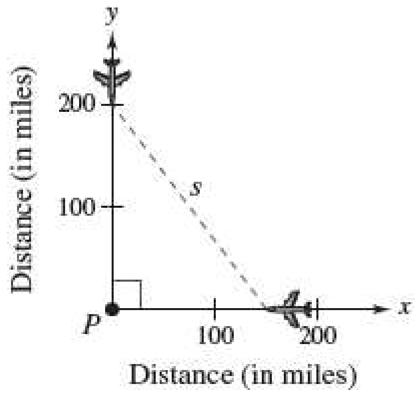
86. Environmental Science The spread of a contaminant is increasing in a circular pattern on the surface of a lake. The radius of the contaminant can be modeled by $r(t) = 5.25\sqrt{t}$, where r is the radius in meters and t is time in hours since contamination.

a. Find a function that gives the area A of the circular leak in terms of the time t since the spread began.

b. Find the size of the contaminated area after 36 hours.

c. Find when the size of the contaminated area is 6250 square meters.

87. Air Traffic Control An air traffic controller spots two planes flying at the same altitude. Their flight paths form a right angle at point P. One plane is 150 miles from point P and is moving at 450 miles per hour. The other plane is 200 miles from point P and is moving at 450 miles per hour. Write the distance s between the planes as a function of time t.



88. Marketing The suggested retail price of a new car is p dollars. The dealership advertised a factory rebate of $\$2000$ and a 9% discount.

    a. Write a function R in terms of p giving the cost of the car after receiving the rebate from the factory.

    b. Write a function S in terms of p giving the cost of the car after receiving the dealership discount.

    c. Form the composite functions $(R \circ S)(p)$ and $(S \circ R)(p)$ and interpret each.

    d. Find $(R \circ S)(24{,}795)$ and $(S \circ R)(24{,}795)$. Which yields the lower cost for the car? Explain.

Conclusions

True or False? In Exercises 89 and 90, determine whether the statement is true or false. Justify your answer.

89. A function that represents the graph of $f(x) = x^2$ shifted three units to the right is $f(g(x))$, where $g(x) = x + 3$.

    True False

90. Given two functions f and g, you can calculate $(f \circ g)(x)$ if and only if the range of g is a subset of the domain of f.

91. Exploration The function in Example 9 can be decomposed in other ways. For which of the following pairs of functions is $h(x) = \dfrac{1}{(x-2)^2}$ equal to $f(g(x))$?

a. $g(x) = \dfrac{1}{x-2}$ and $f(x) = x^2$

b. $g(x) = x^2$ and $f(x) = \dfrac{1}{x-2}$

c. $g(x) = (x-2)^2$ and $f(x) = \dfrac{1}{x}$

92. Proof Prove that the product of two odd functions is an even function, and that the product of two even functions is an even function.

93. Proof Use examples to hypothesize whether the product of an odd function and an even function is even or odd. Then prove your hypothesis.

Exploration In Exercises 94 and 95, three siblings are of three different ages. The oldest is twice the age of the middle sibling, and the middle sibling is six years older than one-half the age of the youngest.

94.

a. Write a composite function that gives the oldest sibling's age in terms of the youngest. Explain how you arrived at your answer.

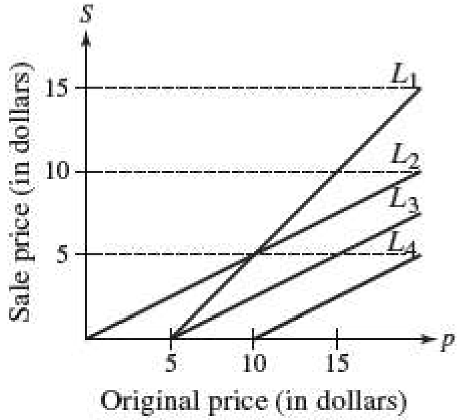
b. The oldest sibling is 16 years old. Find the ages of the other two siblings.

95.

a. Write a composite function that gives the youngest sibling's age in terms of the oldest. Explain how you arrived at your answer.

b. The youngest sibling is two years old. Find the ages of the other two siblings.

96. How Do You See It? The graphs labeled L_1, L_2, L_3, and L_4 represent four different pricing discounts, where p is the original price (in dollars) and S is the sale price (in dollars). Match each function with its graph. Describe the situations in parts (c) and (d).



a. $f(p)$: A 50% discount is applied.

b. $g(p)$: A $\$5$ discount is applied.

c. $(g \circ f)(p)$

d. $(f \circ g)(p)$

Cumulative Mixed Review

Evaluating an Equation In Exercises 97, 98, 99, and 100, find three points that lie on the graph of the equation. (There are many correct answers.)

97. $y = -x^2 + x - 5$

98. $y = \frac{1}{5}x^3 - 4x^2 + 1$

99. $x^2 + y^2 = 49$

100. $y = \frac{x}{x^2 - 5}$