1.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

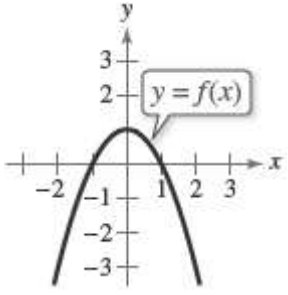
In Exercises 1 and 2, fill in the blank.

1. A function f is _____ on an interval when, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
2. A function f is _____ when, for each x in the domain of f, $f(-x) = f(x)$.
3. The graph of a function f is the segment from $(1, 2)$ to $(4, 5)$, including the endpoints. What is the domain of f?
4. A vertical line intersects a graph twice. Does the graph represent a function?
5. Let f be a function such that $f(2) \geq f(x)$ for all values of x in the interval $(0, 3)$. Does $f(2)$ represent a relative minimum or a relative maximum?
6. Given $f(x) = [\![x]\!]$, in what interval does $f(x) = 5$?

Procedures and Problem Solving

Finding the Domain and Range of a Function In Exercises 7, 8, 9, and 10, use the graph of the function to find the domain and range of f. Then find $f(0)$.

7.



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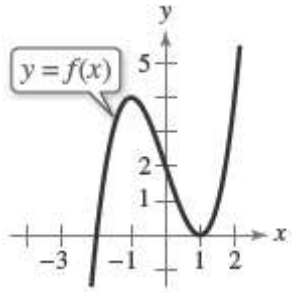

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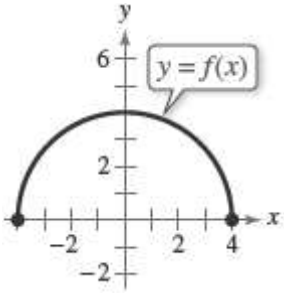

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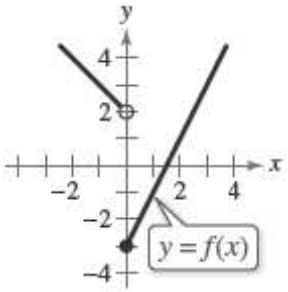


Finding the Domain and Range of a Function In Exercises 11, 12, 13, 14, 15, and 16, use a graphing utility to graph the function and estimate its domain and range. Then find the domain and range algebraically.

11. $f(x) = -2x^2 + 3$

12. $f(x) = x^2 - 1$

13. $f(x) = \sqrt{x + 2}$

14. $h(t) = \sqrt{4 - t^2}$

15. $f(x) = |x + 3|$

16. $f(x) = -\frac{1}{4}|x - 5|$

Analyzing a Graph In Exercises 17 and 18, use the graph of the function to answer the questions.

a. Determine the domain of the function.

b. Determine the range of the function.

c. Find the value(s) of x for which $f(x) = 0$.

d. What are the values of x from part (c) referred to graphically?

e. Find $f(0)$, if possible.

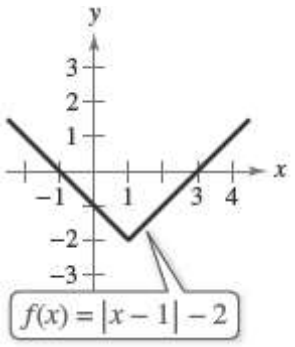
f. What is the value from part (e) referred to graphically?

g. What is the value of f at $x = 1$? What are the coordinates of the point?

h. What is the value of f at $x = -1$? What are the coordinates of the point?

i. The coordinates of the point on the graph of f at which $x = -3$ can be labeled $(-3, f(-3))$ or $(-3,$ $)$.

17.



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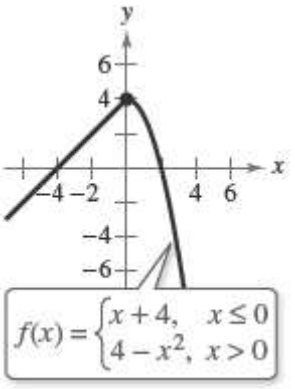
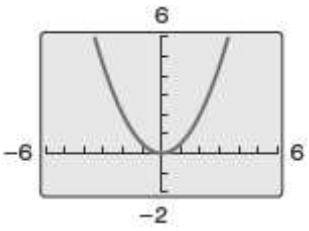
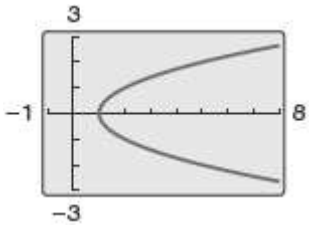


Vertical Line Test for Functions In Exercises 19, 20, 21, and 22, use the Vertical Line Test to determine whether y is a function of x. Describe how you can use a graphing utility to produce the given graph.

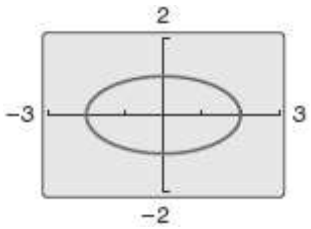
19. $y = \frac{1}{2}x^2$



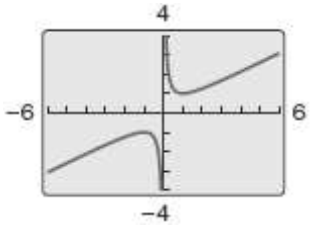
20. $x - y^2 = 1$



21. $0.25x^2 + y^2 = 1$

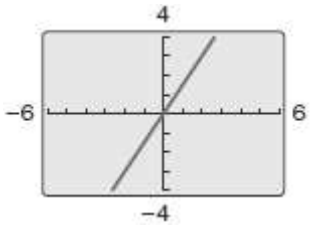


22. $x^2 = 2xy - 1$

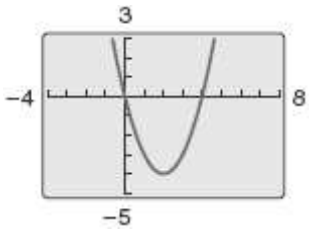


Increasing and Decreasing Functions In Exercises 23, 24, 25, and 26, determine the open intervals on which the function is increasing, decreasing, or constant.

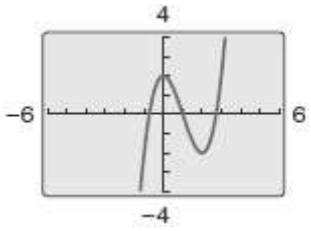
23. $f(x) = \frac{3}{2}x$



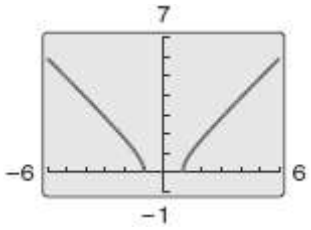
24. $f(x) = x^2 - 4x$



25. $f(x) = x^3 - 3x^2 + 2$



26. $f(x) = \sqrt{x^2 - 1}$



Increasing and Decreasing Functions In Exercises 27, 28, 29, 30, 31, 32, 33, and 34,

a. use a graphing utility to graph the function and

b. determine the open intervals on which the function is increasing, decreasing, or constant.

27. $f(x) = 3$

28. $f(x) = x$

29. $f(x) = x^{2/3}$

30. $f(x) = -x^{3/4}$

31. $f(x) = x\sqrt{x + 3}$

32. $f(x) = x\sqrt{3 - x}$

33. $f(x) = |x + 1| + |x - 1|$

34. $f(x) = -|x + 4| - |x + 1|$

Approximating Relative Minima and Maxima In Exercises 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, and 46, use a graphing utility to graph the function and to approximate any relative minimum or relative maximum values of the function.

35. $f(x) = x^2 - 6x$

36. $f(x) = 3x^2 - 2x - 5$

37. $y = -2x^3 - x^2 + 14x$

38. $y = x^3 - 6x^2 + 15$

39. $h(x) = (x - 1)\sqrt{x}$

40. $g(x) = x\sqrt{4 - x}$

41. $f(x) = x^2 - 4x - 5$

42. $f(x) = 3x^2 - 12x$

43. $f(x) = x^3 - 3x$

44. $f(x) = -x^3 + 3x^2$

45. $f(x) = 3x^2 - 6x + 1$

46. $f(x) = 8x - 4x^2$

Library of Parent Functions In Exercises 47, 48, 49, 50, 51, and 52, sketch the graph of the function by hand. Then use a graphing utility to verify the graph.

47. $f(x) = [\![x]\!] + 2$

48. $f(x) = [\![x]\!] - 3$

49. $f(x) = [\![x - 1]\!] - 2$

50. $f(x) = [\![x + 2]\!] + 1$

51. $f(x) = 2[\![x]\!]$

52. $f(x) = [\![4x]\!]$

Describing a Step Function In Exercises 53 and 54, use a graphing utility to graph the function. State the domain and range of the function. Describe the pattern of the graph.

53. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

54. $g(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)^2$

Sketching a Piecewise-Defined Function In Exercises 55, 56, 57, 58, 59, 60, 61, and 62, sketch the graph of the piecewise-defined function by hand.

55. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$

56. $f(x) = \begin{cases} x + 6, & x \leq -4 \\ 3x - 4, & x > -4 \end{cases}$

57. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$

58. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$

59. $f(x) = \begin{cases} x + 3, & x \leq 0 \\ 3, & 0 < x \leq 2 \\ 2x - 1, & x > 2 \end{cases}$

60. $g(x) = \begin{cases} x + 5, & x \leq -3 \\ 5, & -3 < x < 1 \\ 5x - 4, & x \geq 1 \end{cases}$

61. $f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$

62. $h(x) = \begin{cases} 3 + x, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

Even and Odd Functions In Exercises 63, 64, 65, 66, 67, 68, 69, 70, 71, and 72, use a graphing utility to graph the function and determine whether it is even, odd, or neither.

63. $f(x) = 5$

64. $f(x) = -9$

65. $f(x) = 3x - 2$

66. $f(x) = 4 - 5x$

67. $h(x) = x^2 + 6$

68. $f(x) = -x^2 - 8$

69. $f(x) = \sqrt{1 - x}$

70. $g(t) = \sqrt[3]{t - 1}$

71. $f(x) = |x + 2|$

72. $f(x) = -|x - 5|$

Think about It In Exercises 73, 74, 75, 76, 77, and 78, find the coordinates of a second point on the graph of a function f if the given point is on the graph and the function is

a. even and

b. odd.

73. $\left(\frac{3}{2}, 4\right)$

74. $\left(-\frac{5}{3}, -7\right)$

75. $(-2, -9)$

76. $(5, -1)$

77. $(x, -y)$

78. $(2a, 2c)$

Algebraic-Graphical-Numerical In Exercises 79, 80, 81, 82, 83, 84, 85, and 86, determine whether the function is even, odd, or neither

a. algebraically,

b. graphically by using a graphing utility to graph the function, and

c. numerically by using the *table* feature of the graphing utility to compare $f(x)$ and $f(-x)$ for several values of x.

79. $f(t) = t^2 + 2t - 3$

80. $f(x) = x^6 - 2x^2 + 3$

81. $g(x) = x^3 - 5x$

82. $h(x) = x^5 - 4x^3$

83. $f(x) = x\sqrt{1 - x^2}$

84. $f(x) = x\sqrt{x + 5}$

85. $g(s) = 4s^{2/3}$

86. $f(s) = 4s^{3/5}$

Finding the Intervals Where a Function is Positive In Exercises 87, 88, 89, and 90, graph the function and determine the interval(s) (if any) on the real axis for which $f(x) \geq 0$. Use a graphing utility to verify your results.

87. $f(x) = 4 - x$

88. $f(x) = 4x + 8$

89. $f(x) = x^2 - 9$

90. $f(x) = x^2 - 4x$

91. Business The cost of parking in a metered lot is $\$1.00$ for the first hour and $\$0.50$ for each additional hour or portion of an hour.

    a. A customer needs a model for the cost C of parking in the metered lot for t hours. Which of the following is the appropriate model?

    $$C_1(t) = 1 + 0.50[\![t - 1]\!]$$
    $$C_2(t) = 1 - 0.50[\![-(t - 1)]\!]$$

    b. Use a graphing utility to graph the appropriate model. Estimate the cost of parking in the metered lot for 7 hours and 10 minutes.

92. Why You Should Learn It *(*1.3 Graphs of Functions*)* The cost of sending an overnight package from New York to Atlanta is $\$23.20$ for a package weighing up to but not including 1 pound and $\$2.00$ for each additional pound or portion of a pound. Use the greatest integer function to create a model for the cost C of overnight delivery of a package weighing x pounds, where $x > 0$. Sketch the graph of the function.

Using the Graph of a Function In Exercises 93 and 94, write the height h of the rectangle as a function of x.

93.

94.

95. Modeling Data The number N (in thousands) of existing condominiums and cooperative homes sold each year from 2010 through 2013 in the United States is approximated by the model

$$N = -24.83t^3 + 906t^2 - 10{,}928.2t + 44{,}114, \quad 10 \leq t \leq 13$$

where t represents the year, with $t = 10$ corresponding to 2010. (Source: National Association of Realtors)

a. Use a graphing utility to graph the model over the appropriate domain.

b. Use the graph from part (a) to determine during which years the number of cooperative homes and condos was increasing. During which years was the number decreasing?

c. Approximate the minimum number of cooperative homes and condos sold from 2010 through 2013.

96. Mechanical Engineering The intake pipe of a 100-gallon tank has a flow rate of 10 gallons per minute, and two drain pipes have a flow rate of 5 gallons per minute each. The graph shows the volume V of fluid in the tank as a function of time t. Determine in which pipes the fluid is flowing in specific subintervals of the one-hour interval of time shown on the graph. (There are many correct answers.)

Conclusions

True or False? In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.

97. A function with a square root cannot have a domain that is the set of all real numbers.

    True False

98. It is possible for an odd function to have the interval $[0, \infty)$ as its domain.

Think about It In Exercises 99, 100, 101, 102, 103, and 104, match the graph of the function with the description that best fits the situation.

99.

100.

101.

102.

a. The air temperature at a beach on a sunny day

b. The height of a football kicked in a field goal attempt

c. The number of children in a family over time

d. The population of California as a function of time

e. The depth of the tide at a beach over a 24-hour period

f. The number of cupcakes on a tray at a party

103.

104.

105. Think about It Does the graph in Exercise 19 represent x as a function of y? Explain.

106. Think about It Does the graph in Exercise 21 represent x as a function of y? Explain.

107. Think about It Can you represent the greatest integer function using a piecewise-defined function?

108. Think about It How does the graph of the greatest integer function differ from the graph of a line with a slope of zero?

109. Think about It Let f be an even function. Determine whether g is even, odd, or neither. Explain.

    a. $g(x) = -f(x)$

    b. $g(x) = f(-x)$

    c. $g(x) = f(x) - 2$

    d. $g(x) = -f(x + 3)$

110. How Do You See It? Half of the graph of an odd function is shown.

a. Sketch a complete graph of the function.

b. Find the domain and range of the function.

c. Identify the open intervals on which the function is increasing, decreasing, or constant.

d. Find any relative minimum and relative maximum values of the function.

111. Proof Prove that a function of the following form is odd.

$$y = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$$

112. Proof Prove that a function of the following form is even.

$$y = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$

Cumulative Mixed Review

Identifying Terms and Coefficients In Exercises 113, 114, 115, and 116, identify the terms. Then identify the coefficients of the variable terms of the expression.

113. $-2x^2 + 11x + 3$

114. $10 + 3x$

115. $\frac{x}{3} - 5x^2 + x^3$

116. $7x^4 + \sqrt{2}x^2 - x$

Evaluating a Function In Exercises 117 and 118, evaluate the function at each specified value of the independent variable and simplify.

117. $f(x) = -x^2 - x + 3$

a. $f(4)$

b. $f(-5)$

c. $f(x-2)$

118. $f(x)=x\sqrt{x-3}$

a. $f(3)$

b. $f(12)$

c. $f(6)$

Evaluating a Difference Quotient In Exercises 119 and 120, find the difference quotient and simplify your answer.

119. $f(x)=x^2-2x+9,\ \dfrac{f(3+h)-f(3)}{h},\ h\neq 0$

120. $f(x)=5+6x-x^2,\ \dfrac{f(6+h)-f(6)}{h},\ h\neq 0$