Book Title: Precalculus with Limits: A Graphing Approach, Seventh Edition
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1.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

For instructions on how to use a graphing utility, see Appendix A.

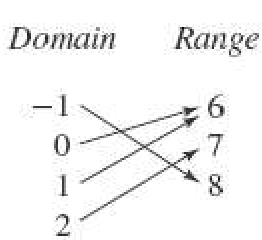
Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

1. A relation that assigns to each element x from a set of inputs, or ____ , exactly one element y in a set of outputs, or ____ , is called a ____ .
2. For an equation that represents y as a function of x, the ____ variable is the set of all x in the domain, and the ____ variable is the set of all y in the range.
3. Can the ordered pairs $(3, 0)$ and $(3, 5)$ represent a function?
4. To find $g(x+1)$, what do you substitute for x in the function $g(x) = 3x - 2$?
5. Does the domain of the function $f(x) = \sqrt{1+x}$ include $x = -2$?
6. Is the domain of a piecewise-defined function *implied* or *explicitly described*?

Procedures and Problem Solving

Testing for Functions In Exercises 7, 8, 9, and 10, does the relation describe a function? Explain your reasoning.

7. 

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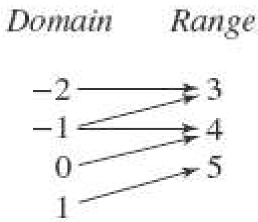

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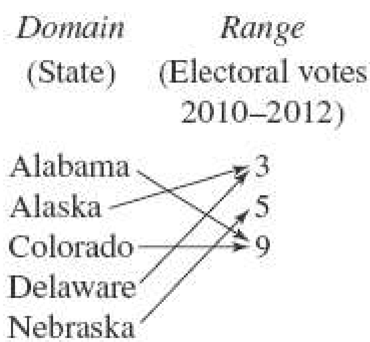


Testing for Functions In Exercises 11, 12, 13, and 14, determine whether the relation represents y as a function of x. Explain your reasoning.

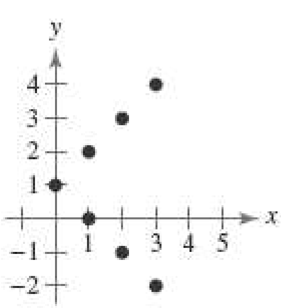
11.

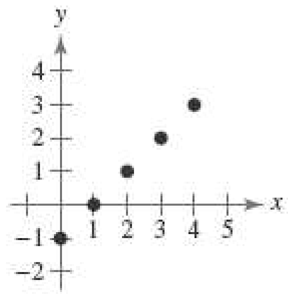
Input, x	−3	−1	0	1	3
Output, y	−9	−1	0	1	−9

12.

Input, x	0	1	2	1	0
Output, y	−4	−2	0	2	4

13.



14. 

Testing for Functions In Exercises 15 and 16, which sets of ordered pairs represent functions from A to B? Explain.

15. $A = \{0, 1, 2, 3\}$ and $B = \{-2, -1, 0, 1, 2\}$

    a. $\{(0, 1), (1, -2), (2, 0), (3, 2)\}$

    b. $\{(0, -1), (2, 2), (1, -2), (3, 0), (1, 1)\}$

    c. $\{(1, 0), (-2, 3), (-1, 3), (0, 0)\}$

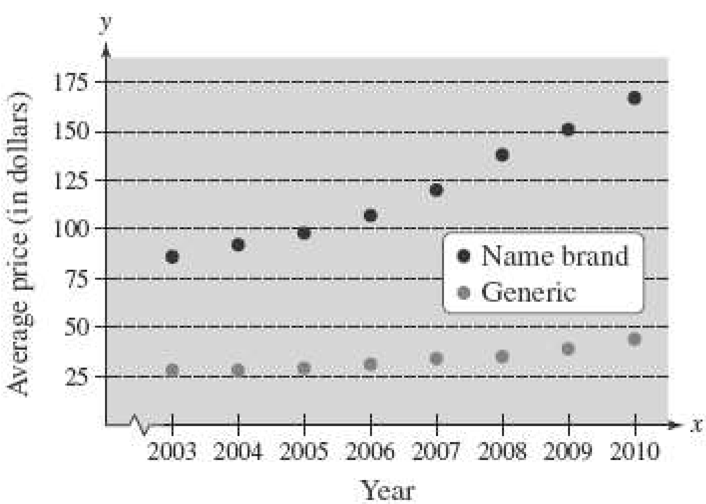
16. $A = \{a, b, c\}$ and $B = \{0, 1, 2, 3\}$

    a. $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$

    b. $\{(a, 1), (b, 2), (c, 3)\}$

    c. $\{(1, a), (0, a), (2, c), (3, b)\}$

Pharmacology In Exercises 17 and 18, use the graph, which shows the average prices of name brand and generic drug prescriptions in the United States. (Source: National Association of Chain Drug Stores)



17. Is the average price of a name brand prescription a function of the year? Is the average price of a generic prescription a function of the year? Explain.

18. Let $b(t)$ and $g(t)$ represent the average prices of name brand and generic prescriptions, respectively, in year t. Find $b(2009)$ and $g(2006)$.

Testing for Functions Represented Algebraically In Exercises 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, and 30, determine whether the equation represents y as a function of x.

19. $x^2 + y^2 = 4$

20. $x = y^2 + 1$

21. $y = \sqrt{x^2 - 1}$

22. $y = \sqrt{x + 5}$

23. $2x + 3y = 4$

24. $x = -y + 5$

25. $y^2 = x^2 - 1$

26. $x + y^2 = 3$

27. $y = |4 - x|$

28. $|y| = 3 - 2x$

29. $x = -7$

30. $y = 8$

Evaluating a Function In Exercises 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, and 46, evaluate the function at each specified value of the independent variable and simplify.

31. $f(t) = 3t + 1$

    a. $f(2)$

    b. $f(-4)$

    c. $f(t + 2)$

32. $g(y) = 7 - 3y$

    a. $g(0)$

    b. $g\left(\frac{7}{3}\right)$

c. $g(s+5)$

33. $h(t)=t^2-2t$

a. $h(2)$

b. $h(1.5)$

c. $h(x-4)$

34. $V(r)=\frac{4}{3}\pi r^3$

a. $V(3)$

b. $V\left(\frac{3}{2}\right)$

c. $V(2r)$

35. $f(y)=3-\sqrt{y}$

a. $f(4)$

b. $f(0.25)$

c. $f(4x^2)$

36. $f(x)=\sqrt{x+8}+2$

a. $f(-4)$

b. $f(8)$

c. $f(x-8)$

37. $q(x)=\frac{1}{x^2-9}$

a. $q(-3)$

b. $q(2)$

c. $q(y+3)$

38. $q(t)=\frac{2t^2+3}{t^2}$

a. $q(2)$

b. $q(0)$

c. $q(-x)$

39. $f(x) = \frac{|x|}{x}$

a. $f(9)$

b. $f(-9)$

c. $f(t)$

40. $f(x) = |x| + 4$

a. $f(5)$

b. $f(-5)$

c. $f(t)$

41. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

a. $f(-1)$

b. $f(0)$

c. $f(2)$

42. $f(x) = \begin{cases} 2x + 5, & x \leq 0 \\ 2 - x, & x > 0 \end{cases}$

a. $f(-2)$

b. $f(0)$

c. $f(1)$

43. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

a. $f(-2)$

b. $f(1)$

c. $f(2)$

44. $f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 1 - 2x^2, & x > 0 \end{cases}$

a. $f(-2)$

b. $f(0)$

c. $f(1)$

45.
$$f(x) = \begin{cases} x+2, & x < 0 \\ 4, & 0 \le x < 2 \\ x^2+1, & x \ge 2 \end{cases}$$

a. $f(-2)$

b. $f(0)$

c. $f(2)$

46.
$$f(x) = \begin{cases} 5-2x, & x < 0 \\ 5, & 0 \le x < 1 \\ 4x+1, & x \ge 1 \end{cases}$$

a. $f(-4)$

b. $f(0)$

c. $f(1)$

Evaluating a Function In Exercises 47, 48, 49, and 50, assume that the domain of f is the set $A = \{-2, -1, 0, 1, 2\}$. Determine the set of ordered pairs representing the function f.

47. $f(x) = (x-1)^2$

48. $f(x) = x^2 - 3$

49. $f(x) = |x| + 2$

50. $f(x) = |x+1|$

Evaluating a Function In Exercises 51 and 52, complete the table.

51. $h(t) = \frac{1}{2}|t+3|$

t	−5	−4	−3	−2	−1
$h(t)$					

52. $f(s) = \dfrac{|s-2|}{s-2}$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$					

Finding the Inputs That Have Outputs of Zero In Exercises 53, 54, 55, and 56, find all values of x such that $f(x) = 0$.

53. $f(x) = 15 - 3x$

54. $f(x) = 5x + 1$

55. $f(x) = \dfrac{9x - 4}{5}$

56. $f(x) = \dfrac{2x - 3}{7}$

Finding the Domain of a Function In Exercises 57, 58, 59, 60, 61, 62, 63, 64, 65, and 66, find the domain of the function.

57. $f(x) = 5x^2 + 2x - 1$

58. $g(x) = 1 - 2x^2$

59. $h(t) = \dfrac{4}{t}$

60. $s(y) = \dfrac{3y}{y + 5}$

61. $f(x) = \sqrt[3]{x - 4}$

62. $f(x) = \sqrt[4]{x^2 + 3x}$

63. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

64. $h(x) = \dfrac{10}{x^2 - 2x}$

65. $g(y) = \dfrac{y + 2}{\sqrt{y - 10}}$

66. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$

Finding the Domain and Range of a Function In Exercises 67, 68, 69, and 70, use a graphing utility to graph the function. Find the domain and range of the function.

67. $f(x) = \sqrt{16 - x^2}$

68. $f(x) = \sqrt{x^2 + 1}$

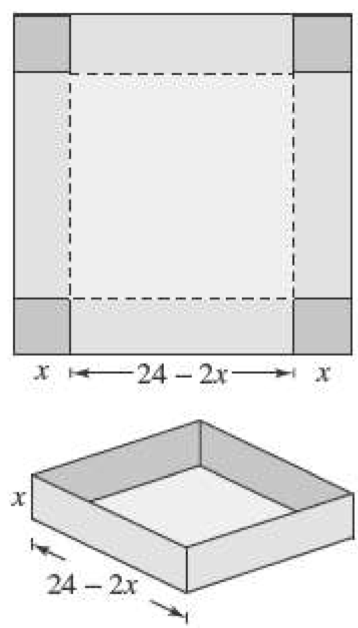
69. $g(x) = |2x + 3|$

70. $g(x) = |3x - 5|$

71. Geometry Write the area A of a circle as a function of its circumference C.

72. Geometry Write the area A of an equilateral triangle as a function of the length s of its sides.

73. Exploration An open box of maximum volume is to be made from a square piece of material, 24 centimeters on a side, by cutting equal squares from the corners and turning up the sides. (See figure.)



a. The table shows the volume V (in cubic centimeters) of the box for various heights x (in centimeters). Use the table to estimate the maximum volume.

Height, x	**Volume, V**
1	484
2	800
3	972

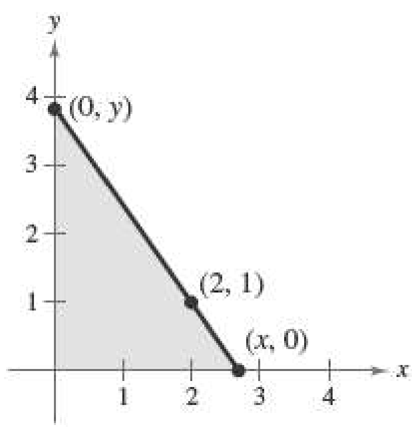
Height, x	Volume, V
4	1024
5	980
6	864

b. Plot the points (x, V) from the table in part (a). Does the relation defined by the ordered pairs represent V as a function of x?

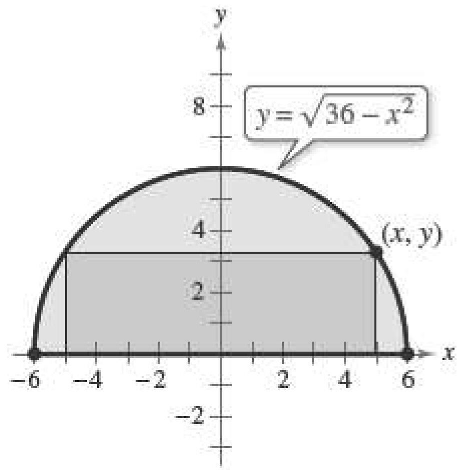
c. If V is a function of x, write the function and determine its domain.

d. Use a graphing utility to plot the points from the table in part (a) with the function from part (c). How closely does the function represent the data? Explain.

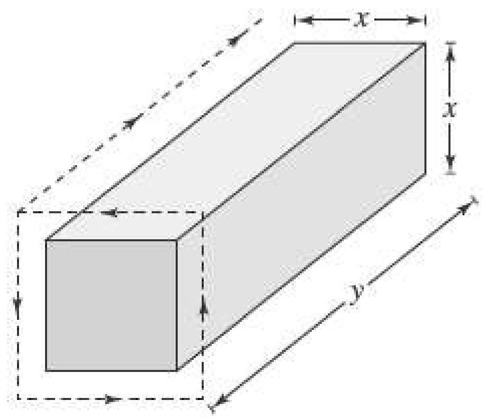
74. Geometry A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point $(2, 1)$, as shown in the figure. Write the area A of the triangle as a function of x and determine the domain of the function.



75. Geometry A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{36 - x^2}$, as shown in the figure. Write the area A of the rectangle as a function of x and determine the domain of the function.



76. Geometry A rectangular package to be sent by the U.S. Postal Service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches. (See figure.)



a. Write the volume V of the package as a function of x. What is the domain of the function?

b. Use a graphing utility to graph the function. Be sure to use an appropriate viewing window.

c. What dimensions will maximize the volume of the package? Explain.

77. Business A company produces a product for which the variable cost is $\$68.75$ per unit and the fixed costs are $\$248{,}000$. The product sells for $\$99.99$. Let x be the number of units produced and sold.

a. The total cost for a business is the sum of the variable cost and the fixed costs. Write the total cost C as a function of the number of units produced.

b. Write the revenue R as a function of the number of units sold.

c. Write the profit P as a function of the number of units sold. (*Note*: $P = R - C$.)

d. Use the model in part (c) to find $P(20{,}000)$. Interpret your result in the context of the situation.

e. Use the model in part (c) to find $P(0)$. Interpret your result in the context of the situation.

78. Modeling Data The table shows the revenue y (in thousands of dollars) of a landscaping business for each month of 2015, with $x = 1$ representing January.

Month, x	**Revenue, y**
1	5.2
2	5.6
3	6.6
4	8.3
5	11.5
6	15.8
7	12.8
8	10.1
9	8.6
10	6.9
11	4.5
12	2.7

The mathematical model below represents the data.

$$f(x) = \begin{cases} -1.97x + 26.3 \\ 0.505x^2 - 1.47x + 6.3 \end{cases}$$

a. Identify the independent and dependent variables and explain what they represent in the context of the problem.

b. What is the domain of each part of the piecewise-defined function? Explain your reasoning.

c. Use the mathematical model to find $f(5)$. Interpret your result in the context of the problem.

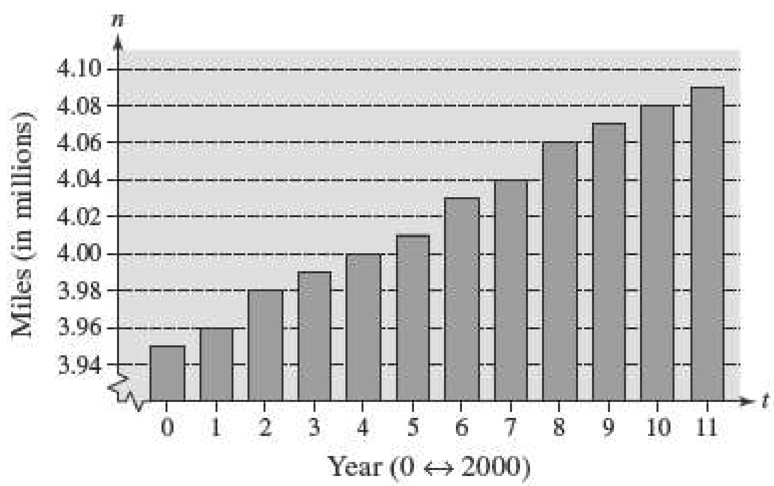
d. Use the mathematical model to find $f(11)$. Interpret your result in the context of the problem.

e. How do the values obtained from the models in parts (c) and (d) compare with the actual data values?

79. Civil Engineering The total numbers n (in millions) of miles for all public roadways in the United States from 2000 through 2011 can be approximated by the model

$$n(t) = \begin{cases} 0.0050t^2 + 0.005t + 3.95, & 0 \le t \le 2 \\ 0.013t + 3.95, & 2 < t \le 11 \end{cases}$$

where t represents the year, with $t = 0$ corresponding to 2000. The actual numbers are shown in the bar graph. (Source: U.S. Federal Highway Administration)



a. Identify the independent and dependent variables and explain what they represent in the context of the problem.

b. Use the *table* feature of a graphing utility to approximate the total number of miles for all public roadways each year from 2000 through 2011.

c. Compare the values in part (b) with the actual values shown in the bar graph. How well does the model fit the data?

d. Do you think the piecewise-defined function could be used to predict the total number of miles for all public roadways for years outside the domain? Explain your reasoning.

80. Why You Should Learn It *(1.2 Functions)* The force F (in tons) of water against the face of a dam is estimated by the function

$$F(y) = 149.76\sqrt{10}y^{5/2}$$

where y is the depth of the water (in feet).

a. Complete the table. What can you conclude?

y	5	10	20	30	40
$F(y)$					

b. Use a graphing utility to graph the function. Describe your viewing window.

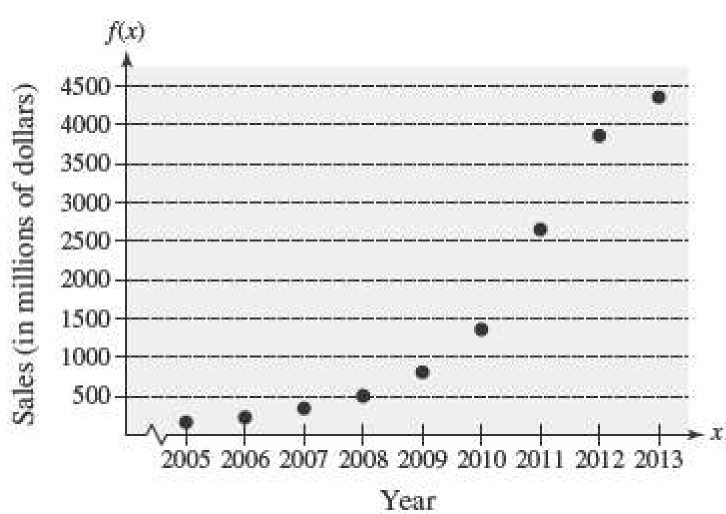
c. Use the table to approximate the depth at which the force against the dam is 1,000,000 tons. Verify your answer graphically. How could you find a better estimate?

81. Projectile Motion The height y (in feet) of a baseball thrown by a child is

$$y = -0.1x^2 + 3x + 6$$

where x is the horizontal distance (in feet) from where the ball was thrown. Will the ball fly over the glove of another child 30 feet away trying to catch the ball? Explain. (Assume that the child who is trying to catch the ball holds a baseball glove at a height of 5 feet.)

82. Business The graph shows the sales (in millions of dollars) of Green Mountain Coffee Roasters from 2005 through 2013. Let $f(x)$ represent the sales in year x. (Source: Green Mountain Coffee Roasters, Inc.)



a. Find $\frac{f(2013) - f(2005)}{2013 - 2005}$ and interpret the result in the context of the problem.

b. An approximate model for the function is

$$S(t) = 90.442t^2 - 1075.25t + 3332.5, \quad 5 \le t \le 13$$

where S is the sales (in millions of dollars) and $t = 5$ represents 2005. Complete the table and compare the results with the data in the graph.

t	5	6	7	8	9	10	11	12	13
$S(t)$									

Evaluating a Difference Quotient In Exercises 83, 84, 85, and 86, find the difference quotient and simplify your answer.

83. $f(x) = 2x, \quad \frac{f(x+c) - f(x)}{c}, \quad c \neq 0$

84. $g(x) = 3x - 1, \quad \frac{g(x+h) - g(x)}{h}, \quad h \neq 0$

85. $f(x) = x^2 - x + 1, \quad \frac{f(2+h) - f(2)}{h}, \quad h \neq 0$

86. $f(x) = x^3 + x, \quad \frac{f(x+h) - f(x)}{h}, \quad h \neq 0$

Conclusions

True or False? In Exercises 87 and 88, determine whether the statement is true or false. Justify your answer.

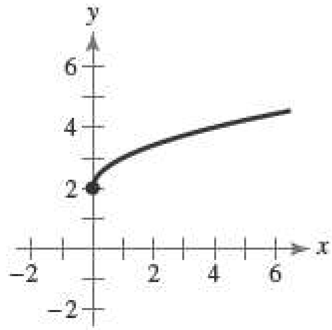
87. The domain of the function $f(x) = x^4 - 1$ is $(-\infty, \infty)$, and the range of f is $(0, \infty)$.

True False

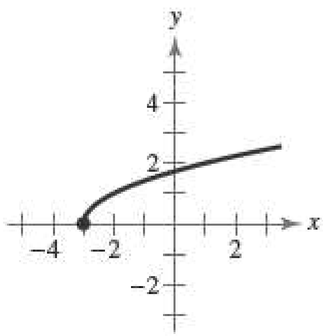
88. The set of ordered pairs $\{(-8, -2), (-6, 0), (-4, 0), (-2, 2), (0, 4), (2, -2)\}$ represents a function.

Think about It In Exercises 89 and 90, write a square root function for the graph shown. Then identify the domain and range of the function.

89.

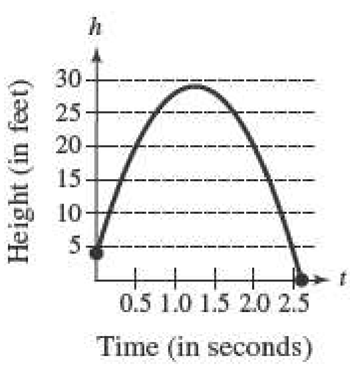


90.



91. Think about It Given $f(x) = x^2$, is f the independent variable? Why or why not?

92. How Do You See It? The graph represents the height h of a projectile after t seconds.



a. Explain why h is a function of t.

b. Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

c. Approximate the domain of h.

d. Is t a function of h? Explain.

Cumulative Mixed Review

Operations with Rational Expressions In Exercises 93, 94, 95, and 96, perform the operation and simplify.

93. $12 - \frac{4}{x+2}$

94. $\frac{3}{x^2 + x - 20} + \frac{2x}{x^2 + 4x - 5}$

95. $\frac{x^5}{2x^3 + 4x^2} \cdot \frac{4x + 8}{3x}$

96. $\frac{x+7}{2(x-9)} \div \frac{x-7}{2(x-9)}$