Identity Crisis

Name:_____

We know that the sine, cosine, and tangent functions all represent ratios of sides in a right triangle. But are they related in any other way? And what do cosecant, secant, and cotangent have to do with it?

A circle of radius 1 is drawn in the first quadrant. Segment DB is tangent to the circle at point A.

1. What is the length of AC?

2. Explain why segment AF represents $\sin\theta$. Which other segment represents $\sin\theta$?

3. Explain why segment CF represents $\cos\theta$. Which other segment represents $\cos\theta$?

4. What is $(\sin\theta)^2 + (\cos\theta)^2$? How do you know?

5. What is the relationship between angles θ and α?

6. Label all the other angles in the diagram with a θ, α, or a 90° marking. Remember from Geometry that a tangent line is always perpendicular to the radius at the point of tangency (Point A).

7. What do you notice about the angles in $\triangle ACF$ and $\triangle BCA$? What does this tell you about the triangles?

STOP
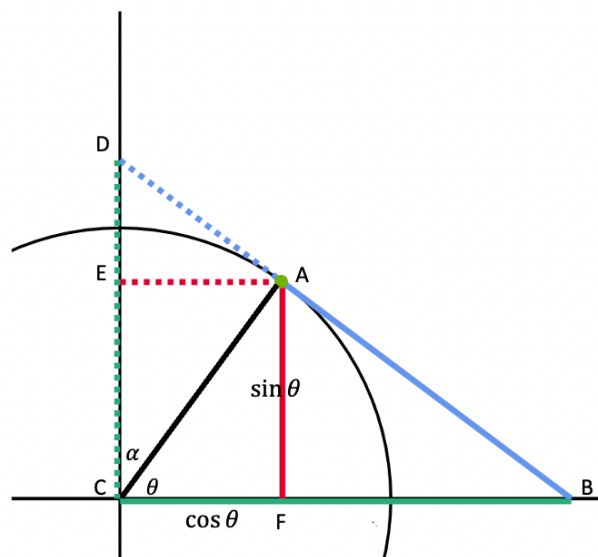
8. Let's see what we can find out about the ratios in these triangles. Fill in the missing part of the proportion.

$$\frac{AF}{FC} = \frac{}{AC}$$

Rewriting this with what we already know we get the proportion below. Which trig ratio should we use to label segment AB?

$$\frac{\sin\theta}{\cos\theta} = \frac{}{1}$$

9. Let's look at another ratio of sides. Fill in the missing parts of the proportion.

$$\frac{AC}{CF} = \frac{BC}{\quad}$$

Let's rewrite this with what we know from the diagram. Which trig ratio should we use to label segment BC?

$$\frac{1}{\cos\theta} = \frac{\quad}{\quad}$$

10. Now let's look at triangles $\triangle AEC$ and $\triangle DAC$. How are these triangles related?

11. Fill in the missing proportions:

$$\frac{EA}{EC} = \frac{AD}{\quad}$$

$$\frac{\cos\theta}{\sin\theta} = \frac{\quad}{1}$$

$$\frac{AC}{EC} = \frac{\quad}{AC}$$

$$\frac{1}{\sin\theta} = \frac{\quad}{1}$$

12. Which segment in the diagram represents $\csc\theta$? Which segment represents $\cot\theta$?

13. What is $1^2 + \tan^2\theta$?

14. What is $1^2 + \cot^2\theta$?

Section 4.9 – Trigonometric Identities

Important Ideas:

Check Your Understanding!

1. If $\sec\theta = \frac{7}{2}$, find $\cos\theta$, $\tan\theta$, $\sin\theta$, $\csc\theta$, and $\cot\theta$.

For questions 2-6, simplify each trig expression to one number or one trig expression.

2. $\tan\theta \cdot \cot\theta =$

3. $\sec^2\theta\,(1 - \sin^2\theta) =$

4. $\dfrac{\cos^2 x + \sin^2 x}{\sec x} =$

5. $\sec^2 x - \tan^2 x =$

6. **Challenge!** $\csc x - \cos x \cot x =$

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